Selected Answers

CHAPTER 1

Section 1.1

Quick Review 1.1

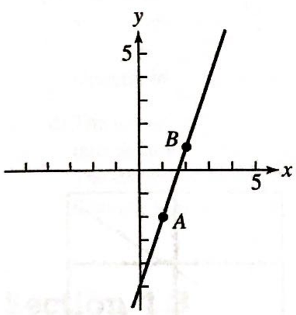
1. -2 **3.** -1 **5.** **(a)** Yes **(b)** No

7. $\sqrt{2}$ **9.** $y = \frac{4}{3}x - \frac{7}{3}$

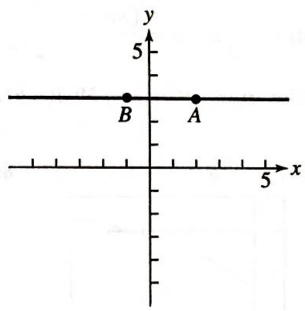
Exercises 1.1

1. $\Delta x = -2, \Delta y = -3$ **3.** $\Delta x = -5, \Delta y = 0$

5. **(a)** and **(c)**, **(b)** 3



7. **(a)** and **(c)**, **(b)** 0



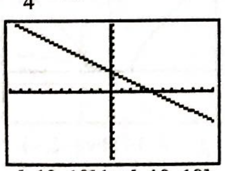
9. $y = 8$ **11.** $y = -4$ **13.** $d = 5.8$ km **15.** $d = 7.4$ km

17. $d = 0.4(t - 6) + 5$ **19.** $y = 1(x - 1) + 1$

21. $y = 2(x - 0) + 3$ **23.** $y = \frac{5}{2}x$

25. **(a)** $-\frac{3}{4}$ **(b)** 3

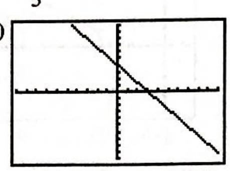
(c)



[−10, 10] by [−10, 10]

27. **(a)** $-\frac{4}{3}$ **(b)** 4

(c)



[−10, 10] by [−10, 10]

29. **(a)** $y = -x$ **(b)** $y = x$ **31.** **(a)** $x = -2$ **(b)** $y = 4$

33. $(3, -5)$ **35.** $(-1, 6)$ **37.** $(1/2, -2)$

39. A burger costs \$4.28 and an order of fries costs \$2.34.

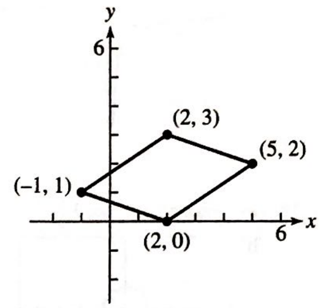
41. **(a)** $k = 2$ **(b)** $k = -2$ **43.** 7:36 PM

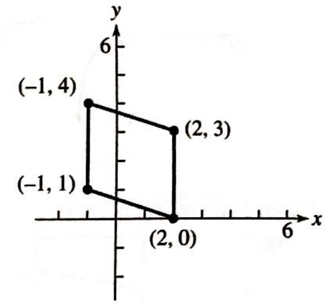
45. False. A vertical line has no slope. **47.** A **49.** D

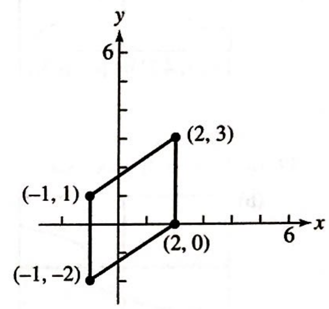
51. $y - 4 = -\frac{3}{4}(x - 3)$ (Notice that the radius has slope $4/3$, and the tangent is perpendicular to the radius.)

53. **(a)** Both algebraic methods lead to solving an equation like $0 = 17$, which is impossible. The conclusion is that there is no pair (x, y) that satisfies both equations simultaneously.
(b) A graph shows the two lines to be parallel.
(c) Two linear equations that are dependent and inconsistent have parallel graphs that do not intersect. Therefore, there is no pair (x, y) that can satisfy both equations simultaneously.

55. The coordinates of the three missing vertices are $(5, 2)$, $(-1, 4)$ and $(-1, -2)$.







57. $y - 6 = \frac{3}{4}(x + 2)$

Section 1.2

Quick Review 1.2

1. $[-2, \infty)$ **3.** $[-1, 7]$ **5.** $(-4, 4)$

7. Translate the graph of f 2 units left and 3 units downward.

9. **(a)** $x = -3, 3$ **(b)** No real solution

11. **(a)** $x = 9$ **(b)** $x = -6$

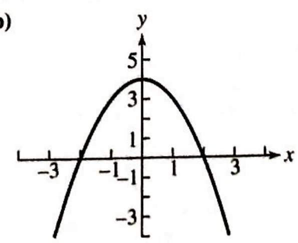
Exercises 1.2

1. **(a)** $A(d) = \pi\left(\frac{d}{2}\right)^2$ **(b)** $A(4) = 4\pi$ in^2

3. **(a)** $S(e) = 6e^2$ **(b)** $S(5) = 150$ ft^2

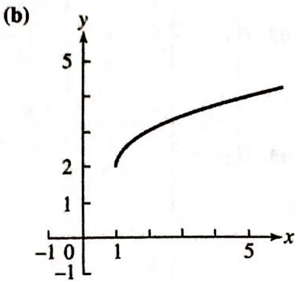
5. **(a)** $(-\infty, \infty)$; $(-\infty, 4]$

(b)



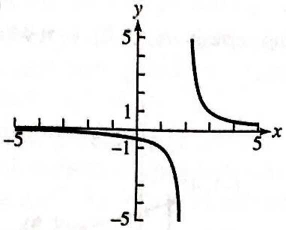
7. **(a)** $[1, \infty)$; $[2, \infty)$

(b)



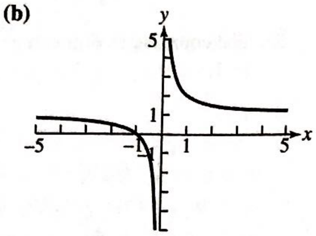
9. **(a)** $(-\infty, 2) \cup (2, \infty)$; $(-\infty, 0) \cup (0, \infty)$

(b)



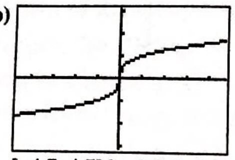
11. **(a)** $(-\infty, 0) \cup (0, \infty)$; $(-\infty, 1) \cup (1, \infty)$

(b)



13. **(a)** $(-\infty, \infty)$; $(-\infty, \infty)$

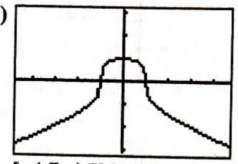
(b)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

15. **(a)** $(-\infty, \infty)$; $(-\infty, 1]$

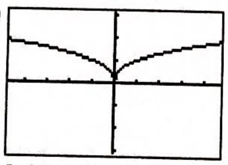
(b)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

17. **(a)** $(-\infty, \infty)$; $[0, \infty)$

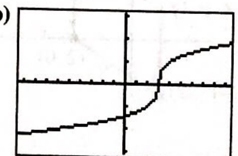
(b)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

19. **(a)** $(-\infty, \infty)$; $(-\infty; \infty)$

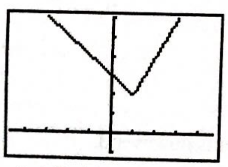
(b)



$[-9.4, 9.4]$ by $[-3.1, 3.1]$

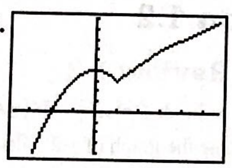
21. Even **23.** Neither **25.** Even **27.** Odd **29.** Neither

31.



$[-4.7, 4.7]$ by $[-1, 6]$

33.



$[-3.7, 5.7]$ by $[-4, 9]$

35. Because if the vertical line test holds, then for each x-coordinate, there is at most one y-coordinate giving a point on the curve. This y-coordinate would correspond to the value assigned to the x-coordinate. Since there's only one y-coordinate, the assignment would be unique.

37. No **39.** Yes

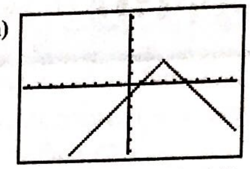
41. $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$

43. $f(x) = \begin{cases} 2 - x, & 0 < x \le 2 \\ \dfrac{5}{3} - \dfrac{x}{3}, & 2 < x \le 5 \end{cases}$

45. $f(x) = \begin{cases} -x, & -1 \le x < 0 \\ 1, & 0 < x \le 1 \\ \dfrac{3}{2} - \dfrac{x}{2}, & 1 < x < 3 \end{cases}$

47. $f(x) = \begin{cases} 0, & 0 \le x \le \dfrac{T}{2} \\ \dfrac{2}{T}x - 1, & \dfrac{T}{2} < x \le T \end{cases}$

49. **(a)**



$[-9.4, 9.4]$ by $[-6.2, 6.2]$

(b) All reals **(c)** $(-\infty, 2]$

51. **(a)** $x^2 + 2$ **(b)** $x^2 + 10x + 22$ **(c)** 2 **(d)** 22 **(e)** -2 **(f)** $x + 10$

53. **(a)** $g(x) = x^2$ **(b)** $g(x) = \dfrac{1}{x - 1}$ **(c)** $f(x) = \dfrac{1}{x}$ **(d)** $f(x) = x^2$

55. **(a)** Because the circumference of the original circle was 8π and a piece of length x was removed.

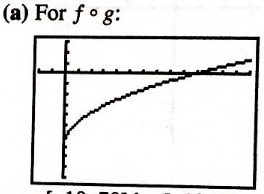
(b) $r = \dfrac{8\pi - x}{2\pi} = 4 - \dfrac{x}{2\pi}$

(c) $h = \sqrt{16 - r^2} = \dfrac{\sqrt{16\pi x - x^2}}{2\pi}$

(d) $V = \dfrac{1}{3}\pi r^2 h = \dfrac{(8\pi - x)^2 \sqrt{16\pi x - x^2}}{24\pi^2}$

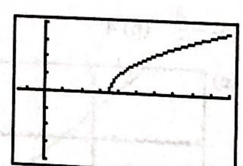
57. False. $f(-x) \ne f(x)$ **59.** B **61.** D

63. **(a)** For $f \circ g$:



$[-10, 70]$ by $[-10, 3]$

Domain: $[0, \infty)$;
Range: $[-7, \infty)$
For $g \circ f$:

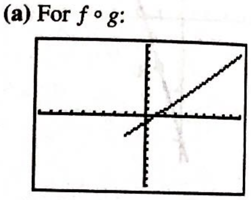


$[-3, 20]$ by $[-4, 4]$

Domain: $[7, \infty)$;
Range: $[0, \infty)$

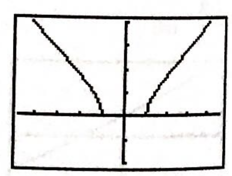
(b) $(f \circ g)(x) = \sqrt{x} - 7$;
$(g \circ f)(x) = \sqrt{x - 7}$

65. **(a)** For $f \circ g$:



$[-10, 10]$ by $[-10, 10]$

Domain: $[-2, \infty)$;
Range: $[-3, \infty)$
For $g \circ f$:

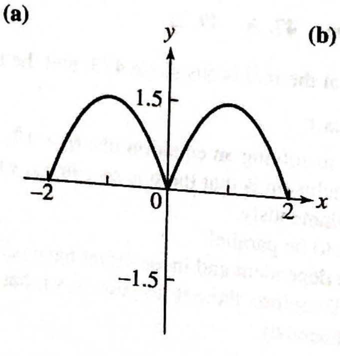


$[-4.7, 4.7]$ by $[-2, 4]$

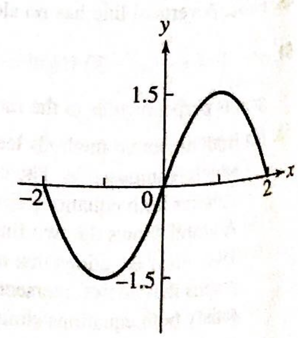
Domain: $(-\infty, -1] \cup [1, \infty)$;
Range: $[0, \infty)$

(b) $(f \circ g)(x) = \left(\sqrt{x + 2}\right)^2 - 3 = x - 1, x \ge -2$
$(g \circ f)(x) = \sqrt{x^2 - 1}$

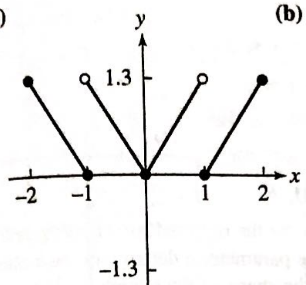
67. **(a)**



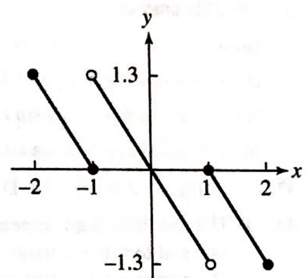
(b)



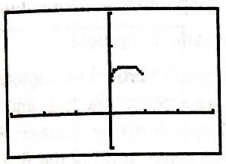
69. (a)



(b)



71. (a)



[−3, 3] by [−1, 3]

(b) Domain of y_1: $[0, \infty)$
Domain of y_2: $(-\infty, 1]$
Domain of y_3: $[0, 1]$

(c) The results for $y_1 - y_2$, $y_2 - y_1$, and $y_1 \cdot y_2$ are the same as for $y_1 + y_2$ above.

Domain of $\frac{y_1}{y_2}$: $[0, 1)$ Domain of $\frac{y_2}{y_1}$: $(0, 1]$

(d) The domain of a sum, difference, or product of two functions is the intersection of their domains.
The domain of a quotient of two functions is the intersection of their domains with any zeros of the denominator removed.

Section 1.3

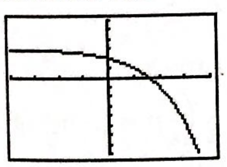
Quick Review 1.3

1. 2.924 **3.** 0.192 **5.** 1.8882

7. \$630.58 **9.** $x^{-18}y^{-5} = \frac{1}{x^{18}y^5}$

Exercises 1.3

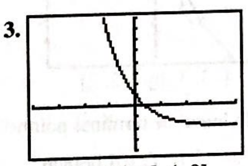
1.



[−4, 4] by [−8, 6]

Domain: All reals
Range: $(-\infty, 3)$

3.



[−4, 4] by [−4, 8]

Domain: All reals
Range: $(-2, \infty)$

5. 3^{4x} **7.** 2^{-6x} **9.** ≈ 2.322 **11.** ≈ -0.631 **13.** (a) **15.** (e)
17. (b) **19.** After 19 years **21.** (a) 63 years (b) 126 years

23. (a) $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$ (b) About 38.1145 days later

25. ≈ 11.433 years **27.** ≈ 11.090 years **29.** ≈ 19.108 years

31. $2^{48} \approx 2.815 \times 10^{14}$

33.

x	y	Δy
1	−1	
		2
2	1	
		2
3	3	
		2
4	5	

35.

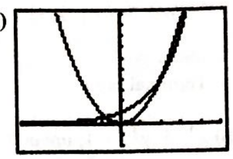
x	y	Δy
1	1	
		3
2	4	
		5
3	9	
		7
4	16	

37. Since $\Delta x = 1$, the corresponding value of Δy is equal to the slope of the line. If the changes in x are constant for a linear function, then the corresponding changes in y are constant as well.

39. $a = 4$ and $b = 3/2$ **41.** False. It is positive $1/9$

43. D **45.** B

47. (a)



[−5, 5] by [−2, 10]

In this window, it appears they cross twice, although a third crossing off-screen appears likely.

(b)

x	change in $Y1$	change in $Y2$
1		
	3	2
2		
	5	4
3		
	7	8
4		

(c) $x = -0.7667, x = 2, x = 4$ (d) $(-0.7667, 2) \cup (4, \infty)$

49. $a = 0.5, k = 3$

Quick Quiz (Sections 1.1–1.3)

1. C **3.** E

Section 1.4

Quick Review 1.4

1. $y = -\frac{5}{3}x + \frac{29}{3}$ **3.** $x = 2$

5. x-intercepts: $x = -4$ and $x = 4$; y-intercepts: none

7. (a) Yes (b) No (c) Yes

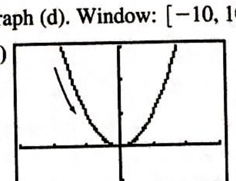
9. (a) $t = \frac{-2x - 5}{3}$ (b) $t = \frac{3y + 1}{2}$

Exercises 1.4

1. Graph (c). Window: $[-4, 4]$ by $[-3, 3]$, $0 \le t \le 2\pi$

3. Graph (d). Window: $[-10, 10]$ by $[-10, 10]$, $0 \le t \le 2\pi$

5. (a)

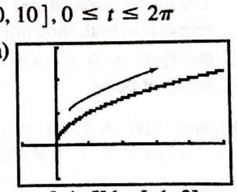


[−3, 3] by [−1, 3]

No initial or terminal point

(b) $y = x^2$; all

7. (a)

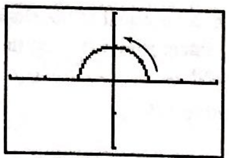


[−1, 5] by [−1, 3]

Initial point: (0, 0)
Terminal point: None

(b) $y = \sqrt{x}$; all (or $x = y^2$; upper half)

9. (a)

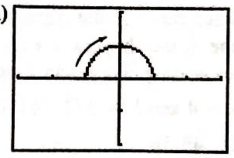


[–3, 3] by [–2, 2]

Initial point: $(1, 0)$
Terminal point: $(-1, 0)$

(b) $x^2 + y^2 = 1$; upper half $\left(\text{or } y = \sqrt{1 - x^2}; \text{ all}\right)$

11. (a)

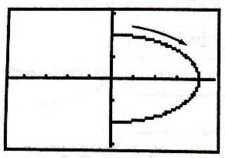


[–3, 3] by [–2, 2]

Initial point: $(-1, 0)$
Terminal point: $(0, 1)$

(b) $x^2 + y^2 = 1$; upper half $\left(\text{or } y = \sqrt{1 - x^2}; \text{ all}\right)$

13. (a)

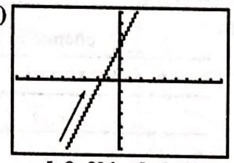


[–4.7, 4.7] by [–3.1, 3.1]

Initial point: $(0, 2)$
Terminal point: $(0, -2)$

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$; right half $\left(\text{or } x = 2\sqrt{4 - y^2}; \text{ all}\right)$

15. (a)

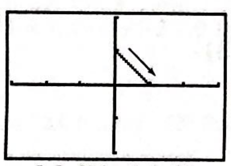


[–9, 9] by [–6, 6]

Initial and terminal point: $(0, 5)$

(b) $y = 2x + 3$; all

17. (a)

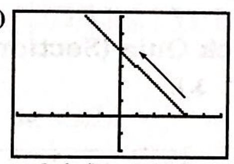


[–3, 3] by [–2, 2]

Initial point: $(0, 1)$
Terminal point: $(1, 0)$

(b) $y = -x + 1$; $(0, 1)$ to $(1, 0)$

19. (a)

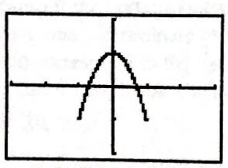


[–6, 6] by [–2, 6]

Initial point: $(4, 0)$
Terminal point: None

(b) $y = -x + 4$; $x \le 4$

21. (a)



[–3, 3] by [–2, 2]

The curve is traced and retraced in both directions, and there is no initial or terminal point.

(b) $y = -2x^2 + 1$; $-1 \le x \le 1$

23. Possible answer: $x = -1 + 5t$, $y = -3 + 4t$, $0 \le t \le 1$

25. Possible answer: $x = t^2 + 1$, $y = t$, $t \le 0$

27. Possible answer: $x = 2 - 3t$, $y = 3 - 4t$, $t \ge 0$

29. $1 < t < 3$ **31.** $-5 \le t < -3$

33. Possible answer: $x = t$, $y = t^2 + 2t + 2$, $t > 0$

35. Possible answers:

(a) $x = a \cos t$, $y = -a \sin t$, $0 \le t \le 2\pi$

(b) $x = a \cos t$, $y = a \sin t$, $0 \le t \le 2\pi$

(c) $x = a \cos t$, $y = -a \sin t$, $0 \le t \le 4\pi$

(d) $x = a \cos t$, $y = a \sin t$, $0 \le t \le 4\pi$

37. False. It is an ellipse. **39.** D **41.** A

43. (a) The resulting graph appears to be the right half of a hyperbola in the first and fourth quadrants. The parameter a determines the x-intercept. The parameter b determines the shape of the hyperbola. If b is smaller, the graph has less steep slopes and appears "sharper." If b is larger, the slopes are steeper and the graph appears more "blunt."

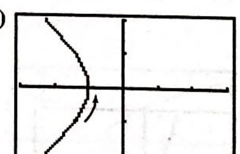
(b) This appears to be the left half of the same hyperbola.

(c) The functions sec t and tan t both approach vertical asymptotes at odd multiples of $\pi/2$, but the limits are $-\infty$ on one side and ∞ on the other. This causes the graph to disappear off one corner of the screen and reappear from the opposite corner, trailing the line in an attempt to keep the graph "connected." For example, as t approaches $\pi/2$ from the left, both sec t and tan t approach $+\infty$, so the graph disappears in quadrant I (the upper-right corner). On the other side of $\pi/2$, both limits are $-\infty$, so the graph reappears in quadrant III (from the lower left corner).

(d) $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = (\sec t)^2 - (\tan t)^2 = 1$ by a standard trigonometric identity.

(e) This changes the orientation of the hyperbola. In this case, b determines the y-intercept of the hyperbola, and a determines the shape. The parameter interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ gives the upper half of the hyperbola. The parameter interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ gives the lower half. The same values of t cause discontinuities and may add extraneous lines to the graph.

45. (a)



[–3, 3] by [–2, 2]

No initial or terminal point

(b) $x^2 - y^2 = 1$; left branch $\left(\text{or } x = -\sqrt{y^2 + 1}; \text{ all}\right)$

47. $x = 2 \cot t$, $y = 2 \sin^2 t$, $0 < t < \pi$

Section 1.5

Quick Review 1.5

1. 1 **3.** $x^{2/3}$

5. Possible answer: $x = t$, $y = \frac{1}{t - 1}$, $t \ge 2$

7. $(4, 5)$ **9. (a)** $(1.58, 3)$ **(b)** No intersection

Exercises 1.5

1. No **3.** Yes **5.** Yes **7.** Yes **9.** No **11.** No

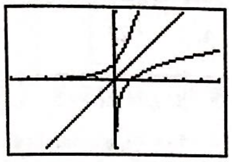
13. $f^{-1}(x) = \frac{x - 3}{2}$ **15.** $f^{-1}(x) = (x + 1)^{1/3}$ or $\sqrt[3]{x + 1}$

17. $f^{-1}(x) = -x^{1/2}$ or $-\sqrt{x}$

19. $f^{-1}(x) = 2 - (-x)^{1/2}$ or $2 - \sqrt{-x}$

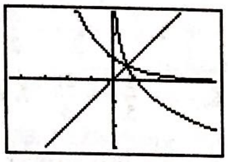
21. $f^{-1}(x) = \dfrac{1}{x^{1/2}}$ or $\dfrac{1}{\sqrt{x}}$ **23.** $f^{-1}(x) = \dfrac{1 - 3x}{x - 2}$

25.



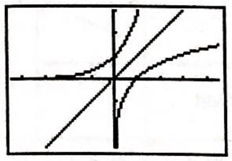
[–6, 6] by [–4, 4]

27.



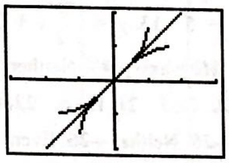
[–4.5, 4.5] by [–3, 3]

29.



[–4.5, 4.5] by [–3, 3]

31.

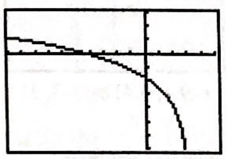


[–3, 3] by [–2, 2]

33. $t = \dfrac{\ln 2}{\ln 1.045} \approx 15.75$ **35.** $x = \ln\left(\dfrac{3 \pm \sqrt{5}}{2}\right) \approx -0.96$ or 0.96

37. $y = e^{2t+4}$

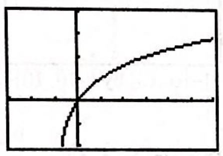
39.



[–10, 5] by [–7, 3]

Domain: $(-\infty, 3)$;
Range: All reals

41.



[–3, 6] by [–2, 4]

Domain: $(-1, \infty)$;
Range: All reals

43. $f^{-1}(x) = \log_2\left(\dfrac{x}{100 - x}\right)$

45. (a) $f(f(x)) = \sqrt{1 - (f(x))^2}$
$= \sqrt{1 - (1 - x^2)}$
$= \sqrt{x^2}$
$= x$, since $x \geq 0$

(b) $f(f(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = x$ for all $x \neq 0$

47. About 14.936 years. (If the interest is only paid annually, it will take 15 years.)

49. (a) All other values of t will make one of the expressions under the radicals negative.

(b) Every point of the form $(\sqrt{2 - t}, \sqrt{2 + t})$ is at distance 4 from the origin.

(c) $(2, 0)$ at $t = -2$ and $(0, 2)$ at $t = 2$.

(d) Both radicals are positive.

(e) The curve is a quarter-circle of radius 4 centered at the origin.

51. (a) Suppose that $f(x_1) = f(x_2)$. Then $mx_1 + b = mx_2 + b$, which gives $x_1 = x_2$ since $m \neq 0$.

(b) $f^{-1}(x) = \dfrac{x - b}{m}$; the slopes are reciprocals.

(c) They are also parallel lines with nonzero slope.

(d) They are also perpendicular lines with nonzero slope.

53. False. Consider $f(x) = x^2$, $g(x) = \sqrt{x}$. Notice that $(f \circ g)(x) = x$ but f is not one-to-one.

55. A **57.** B

59. If the graph of $f(x)$ passes the horizontal line test, so will the graph of $g(x) = -f(x)$, since it's the same graph reflected about the x-axis.

61. (a) Domain: All reals
Range: If $a > 0$, then (d, ∞)
If $a < 0$, then $(-\infty, d)$

(b) Domain: (c, ∞)
Range: All reals

Section 1.6

Quick Review 1.6

1. 60° **3.** $-\dfrac{2\pi}{9}$

5. $x \approx 0.6435, x \approx 2.4981$

7. $x \approx 0.7854\left(\text{or } \dfrac{\pi}{4}\right), x \approx 3.9270\left(\text{or } \dfrac{5\pi}{4}\right)$

9. $f(-x) = (-x)^3 - 3(-x) = -x^3 + 3x$
$= -(x^3 - 3x) = -f(x)$
The graph is symmetric about the origin because if a point (a, b) is on the graph, then so is the point $(-a, -b)$.

Exercises 1.6

1. $\dfrac{5\pi}{4}$ **3.** $\dfrac{1}{2}$ radian or $\approx 28.65°$ **5.** Even **7.** Odd

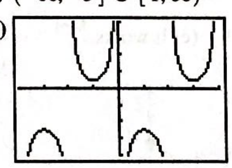
9. $\sin\theta = 8/17, \tan\theta = -8/15, \csc\theta = 17/8,$
$\sec\theta = -17/15, \cot\theta = -15/8$

11. (a) $\dfrac{2\pi}{3}$

(b) $x \neq \dfrac{k\pi}{3}$, for integers k

(c) $(-\infty, -5] \cup [1, \infty)$

(d)



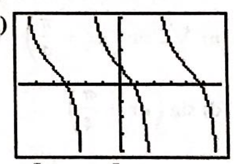
$\left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$ by [–8, 8]

13. (a) $\dfrac{\pi}{3}$

(b) $x \neq \dfrac{k\pi}{6}$, for odd integers k

(c) All reals

(d)



$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by [–8, 8]

15. Possible answers are:

(a) $[0, 4\pi]$ by $[-3, 3]$ **(b)** $[0, 4\pi]$ by $[-3, 3]$

(c) $[0, 2\pi]$ by $[-3, 3]$

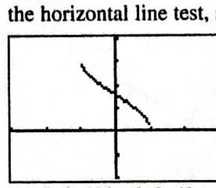
17. (a) π **(b)** 1.5 **(c)** $[-2\pi, 2\pi]$ by $[-2, 2]$

19. (a) π **(b)** 3 **(c)** $[-2\pi, 2\pi]$ by $[-4, 4]$

21. (a) 6 **(b)** 4 **(c)** $[-3, 3]$ by $[-5, 5]$

23. (a) 330 Hz **(b)** E

25. The portion of the curve $y = \cos x$ between $0 \leq x \leq \pi$ passes the horizontal line test, so it is one-to-one.



[–3, 3] by [–2, 4]

27. $\frac{\pi}{6}$ radian or 30° **29.** ≈ -1.3734 radians or $-78.6901°$

31. $x \approx 1.190$ and $x \approx 4.332$

33. $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

35. $x = \frac{7\pi}{6} + 2k\pi$ and $x = \frac{11\pi}{6} + 2k\pi$, k any integer

37. $\cos\theta = \frac{15}{17}$ $\sin\theta = \frac{8}{17}$ $\tan\theta = \frac{8}{15}$

$\sec\theta = \frac{17}{15}$ $\csc\theta = \frac{17}{8}$ $\cot\theta = \frac{15}{8}$

39. $\cos\theta = -\frac{3}{5}$ $\sin\theta = \frac{4}{5}$ $\tan\theta = -\frac{4}{3}$

$\sec\theta = -\frac{5}{3}$ $\csc\theta = \frac{5}{4}$ $\cot\theta = -\frac{3}{4}$

41. $\frac{\sqrt{72}}{11} \approx 0.771$ **43.** $A = -19.75$, $B = \pi/6$, and $C = 60.25$

45. **(a)** $\cot(-x) = \frac{\cos(-x)}{\sin(-x)} = \frac{\cos(x)}{-\sin(x)} = -\cot(x)$

(b) Assume that f is even and g is odd.

Then $\frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f(x)}{g(x)}$ so $\frac{f}{g}$ is odd.

The situation is similar for $\frac{g}{f}$.

47. Assume that f is even and g is odd.
Then $f(-x)\,g(-x) = f(x)[-g(x)] = -f(x)g(x)$ so fg is odd.

49. **(a)** No, 2π **(b)** Yes, π

(c) $y = (\sin x)(\cos x) = \frac{1}{2}\sin 2x$

In general, a product of sinusoids is a sinusoid if they both have the same period.

51. False. The amplitude is 1/2.

53. B **55.** A

57. **(a)** $\sqrt{2}\sin\left(ax + \frac{\pi}{4}\right)$ **(b)** See part (a). **(c)** It works.

(d) $\sin\left(ax + \frac{\pi}{4}\right)$

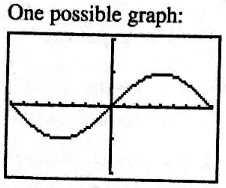
$= \sin(ax) \cdot \frac{1}{\sqrt{2}} + \cos(ax) \cdot \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}(\sin ax + \cos ax)$

So, $\sin(ax) + \cos(ax) = \sqrt{2}\sin\left(ax + \frac{\pi}{4}\right)$.

59. Since $\sin(x)$ has period 2π, $(\sin(x + 2\pi))^3 = (\sin(x))^3$. This function has period 2π. A graph shows that no smaller number works for the period.

61. One possible graph:



$\left[-\frac{\pi}{60}, \frac{\pi}{60}\right]$ by $[-2, 2]$

Quick Quiz (Sections 1.4–1.6)

1. C **3.** E

Review Exercises

1. $y = 3x - 9$ **2.** $y = -\frac{1}{2}x + \frac{3}{2}$ **3.** $x = 0$ **4.** $y = -2x$

5. $y = 2$ **6.** $y = -\frac{2}{5}x + \frac{21}{5}$ **7.** $y = -3x + 3$ **8.** $y = 2x - 5$

9. $y = -\frac{4}{3}x - \frac{20}{3}$ **10.** $y = -\frac{5}{3}x - \frac{19}{3}$ **11.** $y = \frac{2}{3}x + \frac{8}{3}$

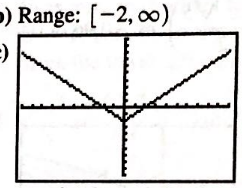
12. $y = \frac{5}{3}x - 5$ **13.** $y = -\frac{1}{2}x + 3$ **14.** $y = -\frac{2}{7}x - \frac{6}{7}$

15. Origin **16.** y-axis **17.** Neither **18.** y-axis

19. Even **20.** Odd **21.** Even **22.** Odd **23.** Odd

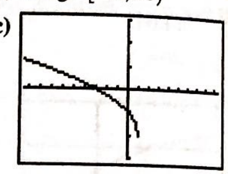
24. Neither **25.** Neither **26.** Even

27. **(a)** Domain: All reals
(b) Range: $[-2, \infty)$
(c)



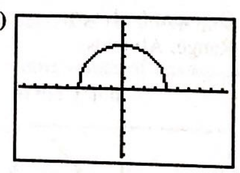
[–10, 10] by [–10, 10]

28. **(a)** Domain: $(-\infty, 1]$
(b) Range $[-2, \infty)$
(c)



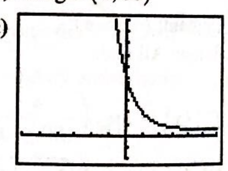
[–9.4, 9.4] by [–3, 3]

29. **(a)** Domain: $[-4, 4]$
(b) Range: $[0, 4]$
(c)



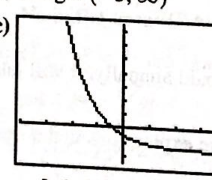
[–9.4, 9.4] by [–6.2, 6.2]

30. **(a)** Domain: All reals
(b) Range: $(1, \infty)$
(c)



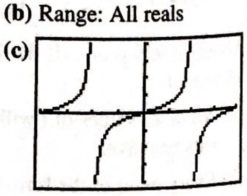
[–6, 6] by [–4, 20]

31. **(a)** Domain: All reals
(b) Range: $(-3, \infty)$
(c)



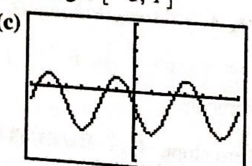
[–4, 4] by [–5, 15]

32. **(a)** Domain: $x \neq \frac{k\pi}{4}$, for odd integers k
(b) Range: All reals
(c)



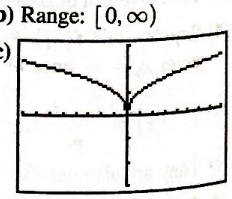
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by $[-8, 8]$

33. **(a)** Domain: All reals
(b) Range: $[-3, 1]$
(c)



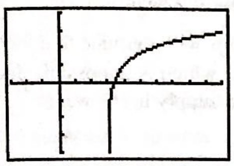
$[-\pi, \pi]$ by $[-5, 5]$

34. **(a)** Domain: All reals
(b) Range: $[0, \infty)$
(c)



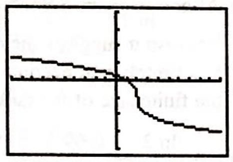
[–8, 8] by [–3, 3]

35. **(a)** Domain: $(3, \infty)$
(b) Range: All reals
(c)



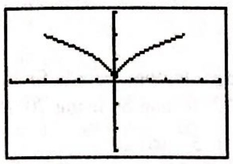
$[-3, 10]$ by $[-4, 4]$

36. **(a)** Domain: All reals
(b) Range: All reals
(c)



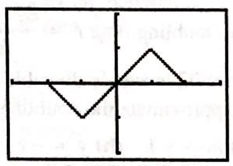
$[-10, 10]$ by $[-4, 4]$

37. **(a)** Domain: $[-4, 4]$
(b) Range: $[0, 2]$
(c)



$[-6, 6]$ by $[-3, 3]$

38. **(a)** Domain: $[-2, 2]$
(b) Range: $[-1, 1]$
(c)



$[-3, 3]$ by $[-2, 2]$

39. $f(x) = \begin{cases} 1 - x, & 0 \le x < 1 \\ 2 - x, & 1 \le x \le 2 \end{cases}$

40. $(x) = \begin{cases} \dfrac{5x}{2}, & 0 \le x < 2 \\ -\dfrac{5}{2}x + 10, & 2 \le x \le 4 \end{cases}$

41. **(a)** 1 **(b)** $\dfrac{1}{\sqrt{2.5}}\left(= \sqrt{\dfrac{2}{5}}\right)$ **(c)** $x, x \ne 0$

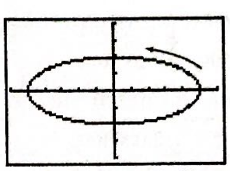
(d) $\dfrac{1}{\sqrt{1/\sqrt{x+2} + 2}}$

42. **(a)** 2 **(b)** 1 **(c)** x **(d)** $\sqrt[3]{\sqrt[3]{x+1} + 1}$

43. **(a)** $(f \circ g)(x) = -x, x \ge -2$
$(g \circ f)(x) = \sqrt{4 - x^2}$
(b) Domain $(f \circ g)$: $[-2, \infty)$
Domain $(g \circ f)$: $[-2, 2]$
(c) Range $(f \circ g)$: $(-\infty, 2]$
Range $(g \circ f)$: $[0, 2]$

44. **(a)** $(f \circ g)(x) = \sqrt[4]{1 - x}$
$(g \circ f)(x) = \sqrt{1 - \sqrt{x}}$
(b) Domain $(f \circ g)$: $(-\infty, 1]$
Domain $(g \circ f)$: $[0, 1]$
(c) Range $(f \circ g)$: $[0, \infty)$
Range $(g \circ f)$: $[0, 1]$

45. **(a)**

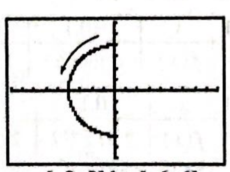


$[-6, 6]$ by $[-4, 4]$

Initial point: $(5, 0)$
Terminal point: $(5, 0)$

(b) $\left(\dfrac{x}{5}\right)^2 + \left(\dfrac{y}{2}\right)^2 = 1$; all

46. **(a)**

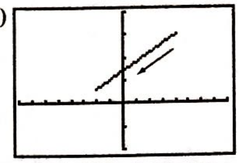


$[-9, 9]$ by $[-6, 6]$

Initial point: $(0, 4)$
Terminal point: $(0, -4)$

(b) $x^2 + y^2 = 16$; left half

47. **(a)**

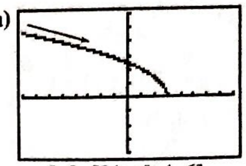


$[-8, 8]$ by $[-10, 20]$

Initial point: $(4, 15)$
Terminal point: $(-2, 3)$

(b) $y = 2x + 7$; from $(4, 15)$ to $(-2, 3)$

48. **(a)**



$[-8, 8]$ by $[-4, 6]$

Initial point: None
Terminal point: $(3, 0)$

(b) $y = \sqrt{6 - 2x}$; all

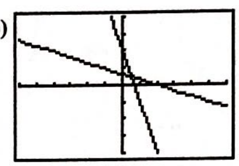
49. Possible answer: $x = -2 + 6t, y = 5 - 2t, 0 \le t \le 1$

50. Possible answer: $x = -3 + 7t, y = -2 + t, -\infty < t < \infty$

51. Possible answer: $x = 2 - 3t, y = 5 - 5t, 0 \le t$

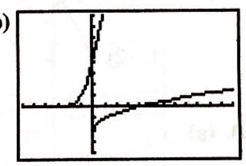
52. Possible answer: $x = t, y = t(t - 4), t \le 2$

53. **(a)** $f^{-1}(x) = \dfrac{2 - x}{3}$
(b)



$[-6, 6]$ by $[-4, 4]$

54. **(a)** $f^{-1}(x) = \sqrt{x} - 2$
(b)



$[-6, 12]$ by $[-4, 8]$

55. ≈ 0.6435 radians or $36.8699°$

56. ≈ -1.1607 radians or $-66.5014°$

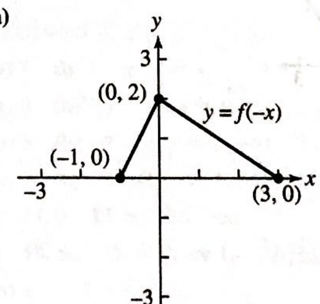
57. $\cos\theta = \dfrac{3}{7}$ $\sin\theta = \dfrac{\sqrt{40}}{7}$ $\tan\theta = \dfrac{\sqrt{40}}{3}$

$\sec\theta = \dfrac{7}{3}$ $\csc\theta = \dfrac{7}{\sqrt{40}}$ $\cot\theta = \dfrac{3}{\sqrt{40}}$

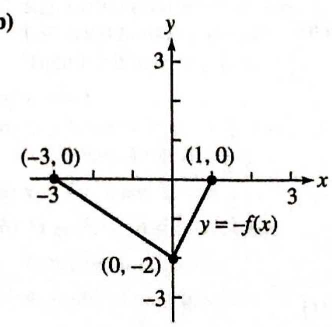
58. **(a)** $x \approx 3.3430$ and $x \approx 6.0818$
(b) $x \approx 3.3430 + 2k\pi$ and $x \approx 6.0818 + 2k\pi$, k any integer

59. $x = -5 \ln 4$

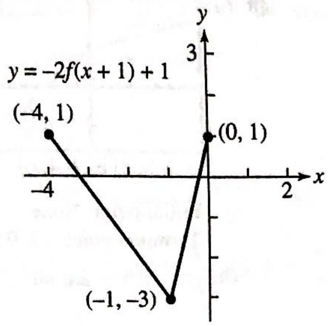
60. **(a)**



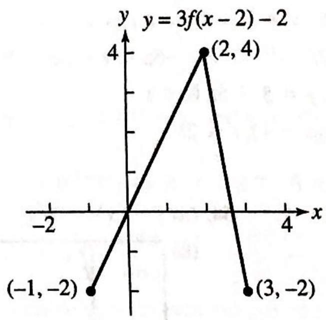
(b)



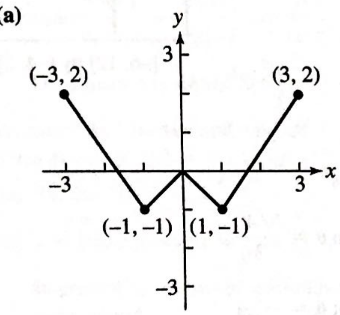
(c)



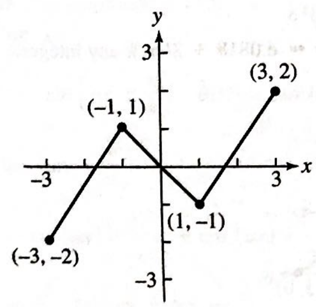
(d)



61. (a)

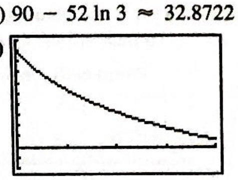


(b)



62. (a) $100{,}000 - 10{,}000x,\ 0 \le x \le 10$
(b) After 4.5 years

63. (a) 90 units
(b) $90 - 52 \ln 3 \approx 32.8722$ units
(c)



[0, 4] by [−20, 100]

64. After $\dfrac{\ln(10/3)}{\ln 1.08} \approx 15.6439$ years
(If the bank pays interest only at the end of the year, it will take 16 years.)

65. (a) $N = 4 \cdot 2^t$ **(b)** 4 days: 64; 1 week: 512
(c) After $\dfrac{\ln 500}{\ln 2} \approx 8.9658$ days, or after nearly 9 days
(d) Because it suggests the number of guppies will continue to double indefinitely and become arbitrarily large, which is impossible due to the finite size of the tank and the oxygen supply in the water.

66. (a) $t = \dfrac{\ln 2}{r} \approx \dfrac{0.69}{r}$
(b) Note that $r = R/100$, so $t = \dfrac{\ln 2}{R/100} = \dfrac{100 \ln 2}{R} \approx \dfrac{69}{R}$
(c) Doubling time $t \approx \dfrac{69 + 1}{R} = \dfrac{70}{R}$

67. Since 72 is evenly divisible by so many integer factors, people find it easier to approximate the doubling time by using $72/R$ than by using $70/R$.

68. (a) $m = -1$ **(b)** $y = -x - 1$ **(c)** $y = x + 3$ **(d)** 2

69. (a) $(2, \infty)$ **(b)** $(-\infty, \infty)$ **(c)** $x = 2 + e \approx 4.718$
(d) $f^{-1}(x) = 2 + e^{1-x}$
(e)
$$\begin{aligned}(f \circ f^{-1})(x) = f(f^{-1}(x)) = f(2 + e^{1-x}) &= 1 - \ln(2 + e^{1-x} - 2)\\ &= 1 - \ln(e^{1-x})\\ &= 1 - (1 - x)\\ &= x\end{aligned}$$
$$\begin{aligned}(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(1 - \ln(x - 2)) &= 2 + e^{1-(1-\ln(x-2))}\\ &= 2 + e^{\ln(x-2)}\\ &= 2 + (x - 2)\\ &= x\end{aligned}$$

70. (a) $(-\infty, \infty)$ **(b)** $[-2, 4]$ **(c)** π **(d)** Even
(e) $x \approx 2.526$

CHAPTER 2

Section 2.1

Quick Review 2.1

1. 0 **3.** 0 **5.** $-4 < x < 4$ **7.** $-1 < x < 5$ **9.** $x - 6$

Exercises 2.1

1. 48 ft/sec **3.** 96 ft/sec

5. $2c^3 - 3c^2 + c - 1$ **7.** $-\dfrac{3}{2}$

9. −15 **11.** 0 **13.** 4

15. (a)

x	−0.1	−0.01	−0.001	−0.0001
$f(x)$	1.566667	1.959697	1.995997	1.999600

(b)

x	0.1	0.01	0.001	0.0001
$f(x)$	2.372727	2.039703	2.003997	2.000400

The limit appears to be 2.

17. (a)

x	−0.1	−0.01	−0.001	−0.0001
$f(x)$	−0.054402	−0.005064	−0.000827	−0.000031

(b)

x	0.1	0.01	0.001	0.0001
$f(x)$	−0.054402	−0.005064	−0.000827	−0.000031

The limit appears to be 0.

19. (a)

x	-0.1	-0.01	-0.001	-0.0001
$f(x)$	2.0567	2.2763	2.2999	2.3023

(b)

x	0.1	0.01	0.001	0.0001
$f(x)$	2.5893	2.3293	2.3052	2.3029

The limit appears to be approximately 2.3.

21. Expression not defined at $x = -2$. There is no limit.

23. Expression not defined at $x = 0$. There is no limit.

25. $\frac{1}{2}$ 27. $-\frac{1}{2}$ 29. 12 31. -1 33. 0

35. Answers will vary. One possible graph is given by the window $[-4.7, 4.7]$ by $[-15, 15]$ with Xscl = 1 and Yscl = 5.

37. 0 39. 0 41. 1

43. (a) True (b) True (c) False (d) True (e) True (f) True (g) False (h) False (i) False (j) False

45. (a) 3 (b) -2 (c) No limit (d) 1

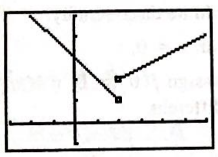
47. (a) -4 (b) -4 (c) -4 (d) -4

49. (a) 4 (b) -3 (c) No limit (d) 4

51. (c) 53. (d)

55. (a) 6 (b) 0 (c) 9 (d) -3

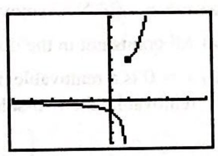
57. (a)



[–3, 6] by [–1, 5]

(b) Right-hand: 2 Left-hand: 1
(c) No, because the two one-sided limits are different

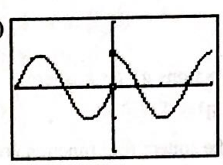
59. (a)



[–5, 5] by [–4, 8]

(b) Right-hand: 4
Left-hand: no limit
(c) No, because the left-hand limit doesn't exist

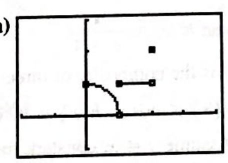
61. (a)



[–2π, 2π] by [–2, 2]

(b) $(-2\pi, 0) \cup (0, 2\pi)$
(c) $c = 2\pi$ (d) $c = -2\pi$

63. (a)



[–2, 4] by [–1, 3]

(b) $(0, 1) \cup (1, 2)$
(c) $c = 2$ (d) $c = 0$

65. 0 67. 0 69. (a) 14.7 m/sec (b) 29.4 m/sec

71. True. Definition of limit. 73. C 75. E

77. (a) Because the right-hand limit at zero depends only on the values of the function for positive x-values near zero

(b) Use: area of triangle $= \left(\frac{1}{2}\right)(\text{base})(\text{height})$

area of circular sector $= \dfrac{(\text{angle})(\text{radius})^2}{2}$

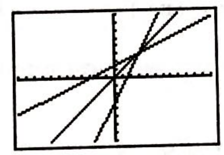
(c) This is how the areas of the three regions compare.
(d) Multiply by 2 and divide by $\sin\theta$.
(e) Take reciprocals, remembering that all of the values involved are positive.
(f) The limits for $\cos\theta$ and 1 are both equal to 1. Since $\frac{\sin\theta}{\theta}$ is between them, it must also have a limit of 1.
(g) $\dfrac{\sin(-\theta)}{-\theta} = \dfrac{-\sin(\theta)}{-\theta} = \dfrac{\sin(\theta)}{\theta}$
(h) If the function is symmetric about the y-axis, and the right-hand limit at zero is 1, then the left-hand limit at zero must also be 1.
(i) The two one-sided limits both exist and are equal to 1.

79. (a) $f\left(\frac{\pi}{6}\right) = \frac{1}{2}$
(b) One possible answer: $a = 0.305, b = 0.775$
(c) One possible answer: $a = 0.513, b = 0.535$

Section 2.2

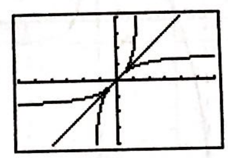
Quick Review 2.2

1. $f^{-1}(x) = \dfrac{x + 3}{2}$



[–12, 12] by [–8, 8]

3. $f^{-1}(x) = \tan(x), -\frac{\pi}{2} < x < \frac{\pi}{2}$



[–6, 6] by [–4, 4]

5. $q(x) = \frac{2}{3}$

$r(x) = -3x^2 - \left(\frac{5}{3}\right)x + \frac{7}{3}$

7. (a) $f(-x) = \cos x$ (b) $f\left(\frac{1}{x}\right) = \cos\left(\frac{1}{x}\right)$

9. (a) $f(-x) = -\dfrac{\ln|x|}{x}$ (b) $f\left(\frac{1}{x}\right) = -x\ln|x|$

Exercises 2.2

1. (a) 1 (b) 1 (c) $y = 1$

3. (a) 0 (b) $-\infty$ (c) $y = 0$

5. (a) 3 (b) -3 (c) $y = 3, y = -3$

7. (a) 1 (b) -1 (c) $y = 1, y = -1$

9. 0 11. 0 13. ∞ 15. $-\infty$

17. 0 19. ∞ 21. Both are 1. 23. Both are 1. 25. Both are -2.

27. (a) $x = -2, x = 2$
(b) Left-hand limit at -2 is ∞.
Right-hand limit at -2 is $-\infty$.
Left-hand limit at 2 is $-\infty$.
Right-hand limit at 2 is ∞.

29. (a) $x = -1$
(b) Left-hand limit at -1 is $-\infty$.
Right-hand limit at -1 is ∞.

31. (a) $x = k\pi$, k any integer
(b) At each vertical asymptote:
Left-hand limit is $-\infty$.
Right-hand limit is ∞.

33. Vertical asymptotes at $a = (4k + 1)\frac{\pi}{2}$ and $b = (4k + 3)\frac{\pi}{2}$, k any integer.
$\lim_{x\to a^-} f(x) = \infty$, $\lim_{x\to a^+} f(x) = -\infty$, $\lim_{x\to b^-} f(x) = -\infty$, $\lim_{x\to b^+} f(x) = \infty$

35. (a) **37.** (d) **39.** **(a)** $3x^2$ **(b)** None

41. **(a)** $\frac{1}{2x}$ **(b)** $y = 0$ **43.** **(a)** $4x^2$ **(b)** None

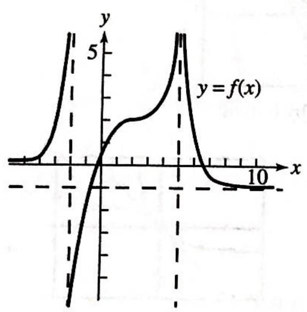
45. **(a)** e^x **(b)** $-2x$ **47.** **(a)** x **(b)** x

49. At ∞: ∞ At $-\infty$: 0

51. At ∞: 0 At $-\infty$: 0

53. **(a)** 0 **(b)** -1 **(c)** $-\infty$ **(d)** -1

55. One possible answer:



57. $\frac{f_1(x)/f_2(x)}{g_1(x)/g_2(x)} = \frac{f_1(x)/g_1(x)}{f_2(x)/g_2(x)}$. As x goes to infinity, $\frac{f_1}{g_1}$ and $\frac{f_2}{g_2}$ both approach 1. Therefore, using the above equation, $\frac{f_1/f_2}{g_1/g_2}$ must also approach 1.

59. True. For example, $f(x) = \frac{x}{\sqrt{x^2+1}}$ has $y = \pm 1$ as horizontal asymptotes.

61. A **63.** C

65. **(a)** $f \to -\infty$ as $x \to 0^-$, $f \to \infty$ as $x \to 0^-$, $g \to 0$, $fg \to 1$
(b) $f \to \infty$ as $x \to 0^-$, $f \to -\infty$ as $x \to 0^+$, $g \to 0$, $fg \to -8$
(c) $f \to -\infty$ as $x \to 2^-$, $f \to \infty$ as $x \to 2^+$, $g \to 0$, $fg \to 0$
(d) $x \to \infty$, $g \to 0$, $fg \to \infty$
(e) Nothing—you need more information to decide.

67. For $x > 0$, $0 < e^{-x} < 1$, so $0 < \frac{e^{-x}}{x} < \frac{1}{x}$.
Since both 0 and $\frac{1}{x}$ approach zero as $x \to \infty$, the Squeeze Theorem states that $\frac{e^{-x}}{x}$ must also approach zero.

69. Limit $= 2$, because $\frac{\ln x^2}{\ln x} = \frac{2\ln x}{\ln x} = 2$.

71. Limit $= 1$. Since

$$\ln(x+1) = \ln x\left(1 + \frac{1}{x}\right) = \ln x + \ln\left(1 + \frac{1}{x}\right),$$

$$\frac{\ln(x+1)}{\ln x} = \frac{\ln x + \ln(1 + 1/x)}{\ln x} = 1 + \frac{\ln(1 + 1/x)}{\ln x}.$$

But as $x \to \infty$, $1 + \frac{1}{x}$ approaches 1, so $\ln\left(1 + \frac{1}{x}\right)$ approaches $\ln(1) = 0$. Also, as $x \to \infty$, $\ln x$ approaches infinity. This means the second term above approaches 0 and the limit is 1.

Quick Quiz (Sections 2.1 and 2.2)

1. D **3.** E

Section 2.3

Quick Review 2.3

1. 2 **3.** **(a)** 1 **(b)** 2 **(c)** No limit **(d)** 2

5. $g(x) = \sin x, x \geq 0$ $(f \circ g)(x) = \sin^2 x, x \geq 0$

7. $x = \frac{1}{2}, -5$ **9.** $x = 1$

Exercises 2.3

1. $x = -2$, infinite discontinuity **3.** None

5. All points not in the domain, i.e., all $x < -3/2$

7. $x = 0$, jump discontinuity **9.** $x = 0$, infinite discontinuity

11. **(a)** Yes **(b)** Yes **(c)** Yes **(d)** Yes **13.** **(a)** No **(b)** No **15.** 0

17. No, because the right-hand and left-hand limits are not the same at zero

19. **(a)** $x = 2$ **(b)** Not removable; the one-sided limits are different.

21. **(a)** $x = 1$ **(b)** Not removable; it's an infinite discontinuity.

23. **(a)** All points not in the domain along with $x = 0, 1$
(b) $x = 0$ is a removable discontinuity; assign $f(0) = 0$, $x = 1$ is not removable; the two-sided limits are different.

25. $y = x - 3$ **27.** $y = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ **29.** $y = \sqrt{x} + 2$

31. The domain of f is all real numbers $x \neq 3$. f is continuous at all those points, so f is a continuous function.

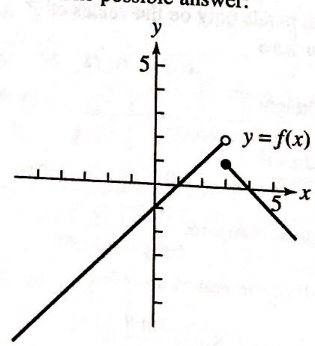
33. f is the composite of two continuous functions $g \circ h$ where $g(x) = \sqrt{x}$ and $h(x) = \frac{x}{x+1}$.

35. f is the composite of three continuous functions $g \circ h \circ k$ where $g(x) = \cos x$, $h(x) = \sqrt[3]{x}$, and $k(x) = 1 - x$.

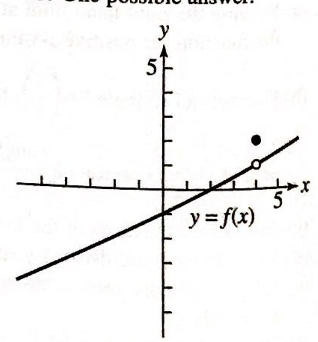
37. Assume $y = x$, constant functions, and the square root function are continuous.
Use the sum, composite, and quotient theorems.
Domain: $(-2, \infty)$

39. Assume $y = x$ and the absolute value function are continuous.
Use the product, constant multiple, difference, and composite theorems.
Domain: $(-\infty, \infty)$

41. One possible answer:



43. One possible answer:

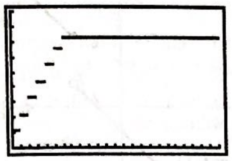


45. $x \approx -0.724$ and $x \approx 1.221$ **47.** $a = \frac{4}{3}$ **49.** $a = 4$

51. Consider $f(x) = x - e^{-x}$, f is continuous, $f(0) = -1$, and $f(1) = 1 - \frac{1}{e} > 0.5$. By the Intermediate Value Theorem, for some c in $(0, 1)$, $f(c) = 0$ and $e^{-c} = c$.

53. (a) $f(x) = \begin{cases} -1.10 \text{ int}(-x), & 0 \le x \le 6 \\ 7.25, & 6 < x \le 24 \end{cases}$

(b)



[0, 24] by [0, 9]

This is continuous at all values of x in the domain $[0, 24]$ except for $x = 0, 1, 2, 3, 4, 5, 6$.

55. False. If f has a jump discontinuity at $x = a$, then $\lim_{x\to a^-} f(x) \ne \lim_{x\to a} f(x)$ so f is not continuous at $x = a$.

57. E **59.** E

61. This is because $\lim_{h\to 0} f(a + h) = \lim_{x\to a} f(x)$.

63. Since the absolute value function is continuous, this follows from the theorem about continuity of composite functions.

Section 2.4

Quick Review 2.4

1. $\Delta x = 8, \Delta y = 3$ **3.** Slope $= -\frac{4}{7}$ **5.** $y = \frac{3}{2}x + 6$

7. $y = -\frac{3}{4}x + \frac{19}{4}$ **9.** $y = -\frac{2}{3}x + \frac{7}{3}$

Exercises 2.4

1. (a) 19 **(b)** 1

3. (a) $\frac{1 - e^{-2}}{2} \approx 0.432$ **(b)** $\frac{e^3 - e}{2} \approx 8.684$

5. (a) $-\frac{4}{\pi} \approx -1.273$ **(b)** $-\frac{3\sqrt{3}}{\pi} \approx -1.654$

7. Using $Q_1 = (10, 225)$, $Q_2 = (14, 375)$, $Q_3 = (16.5, 475)$, $Q_4 = (18, 550)$, and $P = (20, 650)$

(a)

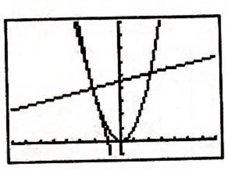
Secant	Slope
PQ_1	43
PQ_2	46
PQ_3	50
PQ_4	50

Units are meters/second.

(b) Approximately 50 m/sec

9. (a) -4 **(b)** $y = -4x - 4$ **(c)** $y = \frac{1}{4}x + \frac{9}{2}$

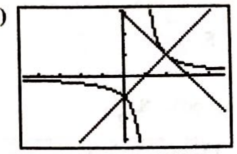
(d)



[-8, 7] by [-1, 9]

11. (a) -1 **(b)** $y = -x + 3$ **(c)** $y = x - 1$

(d)



[-4.7, 4.7] by [-3.1, 3.1]

13. (a) 1 **(b)** -1

15. No. Slope from the left is -2; slope from the right is 2. The two-sided limit of the difference quotient doesn't exist.

17. Yes. The slope is $-\frac{1}{4}$.

19. (a) $2a$

(b) The slope of the tangent steadily increases as a increases.

21. (a) $-\frac{1}{(a - 1)^2}$

(b) The slope of the tangent is always negative. The tangents are very steep near $x = 1$ and nearly horizontal as a moves away from the origin.

23. 3 ft/sec **25.** $-1/4$ ft/sec **27.** 19.6 m/sec

29. 6π in²/in. **31.** 3.72 m/sec **33.** $(-2, -5)$

35. (a) At $x = 0$: $y = -x - 1$

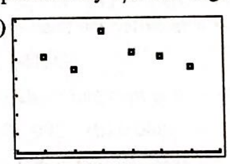
At $x = 2$: $y = -x + 3$

(b) At $x = 0$: $y = x - 1$

At $x = 2$: $y = x - 1$

37. $-4/9$ degrees per mg. Additional dosage ΔD will drop temperature by approximately $4/9\ \Delta D$ degrees.

39. (a)



[2007, 2014] by [0, 1400]

(b) Slope of $PQ_1 = 12$, slope of $PQ_2 = -2$, slope of $PQ_3 = -23$.

41. True. The normal line is perpendicular to the tangent line at the point.

43. D **45.** C

47. (a) $\frac{e^{1+h} - e}{h}$ **(b)** Limit ≈ 2.718

(c) They're about the same.

(d) Yes, it has a tangent whose slope is about e.

49. No **51.** Yes

53. This function has a tangent with slope zero at the origin. It is squeezed between two functions, $y = x^2$ and $y = -x^2$, both of which have slope zero at the origin.

Looking at the difference quotient, $-h \le \frac{f(0 + h) - f(0)}{h} \le h$,

so the Squeeze Theorem tells us that the limit is 0.

55. Slope ≈ 0.540

57. If $x = a + h$, then $x - a = h$. Replacing $f(a + h)$ by $f(x)$ and h by $x - a$ turns the first expression given for the difference quotient into the second expression.

Quick Quiz (Sections 2.3 and 2.4)

1. D **3.** B

Review Exercises

1. -15 **2.** $\frac{5}{21}$ **3.** No limit **4.** No limit **5.** $-\frac{1}{4}$

6. $\frac{2}{5}$ **7.** $+\infty$ (as $x \to +\infty$), $-\infty$ (as $x \to -\infty$) **8.** $\frac{1}{2}$

9. 2 **10.** 0 **11.** 6 **12.** 5 **13.** 0 **14.** 1 **15.** Limit exists

16. Limit exists **17.** Limit exists **18.** Doesn't exist

19. Limit exists **20.** Limit exists **21.** Yes **22.** No

23. No **24.** Yes

25. (a) 1 **(b)** 1.5 **(c)** No

(d) g is discontinuous at $x = 3$ (and points not in domain).

(e) Yes, can remove discontinuity at $x = 3$ by assigning the value 1 to $g(3)$.

26. (a) 1.5 **(b)** 0 **(c)** 0 **(d)** No

(e) k is discontinuous at $x = 1$ (and points not in domain).

(f) Discontinuity at $x = 1$ is not removable because the two one-sided limits are different.

27. (a) Vertical Asymp.: $x = -2$

(b) Left-hand limit $= -\infty$

Right-hand limit $= \infty$

28. (a) Vertical Asymp.: $x = 0$ and $x = -2$

(b) At $x = 0$:

Left-hand limit $= -\infty$

Right-hand limit $= -\infty$

At $x = -2$:

Left-hand limit $= -\infty$

Right-hand limit $= -\infty$

29. (a) At $x = -1$:

Left-hand limit $= 1$

Right-hand limit $= 1$

At $x = 0$:

Left-hand limit $= 0$

Right-hand limit $= 0$

At $x = 1$:

Left-hand limit $= -1$

Right-hand limit $= 1$

(b) At $x = -1$:

Yes, the limit is 1.

At $x = 0$:

Yes, the limit is 0.

At $x = 1$:

No, the limit doesn't exist because the two one-sided limits are different.

(c) At $x = -1$:

Continuous because $f(-1) =$ the limit.

At $x = 0$:

Discontinuous because $f(0) \neq$ the limit.

At $x = 1$:

Discontinuous because limit doesn't exist.

30. (a) Left-hand limit $= 3$ Right-hand limit $= -3$

(b) No, because the two one-sided limits are different

(c) Every place except for $x = 1$

(d) At $x = 1$

31. $x = -2$ and $x = 2$

32. There are no points of discontinuity.

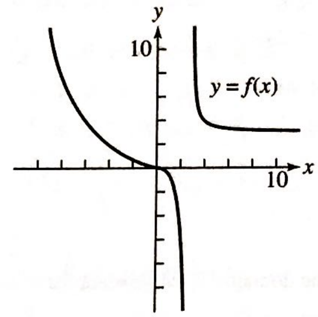
33. (a) $2/x$ **(b)** $y = 0$ (x-axis) **34. (a)** 2 **(b)** $y = 2$

35. (a) x^2 **(b)** None **36. (a)** x **(b)** None

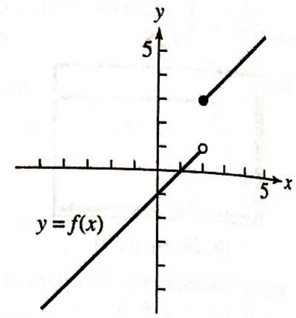
37. (a) e^x **(b)** x **38. (a)** $\ln|x|$ **(b)** $\ln|x|$

39. $k = 8$ **40.** $k = \frac{1}{2}$

41. One possible answer:



42. One possible answer:



43. $\frac{2}{\pi}$ **44.** $\frac{2}{3}\pi aH$ **45.** $12a$ **46.** $2a - 1$

47. (a) -1 **(b)** $y = -x - 1$ **(c)** $y = x - 3$ **48.** $\left(\frac{3}{2}, -\frac{9}{4}\right)$

49. 0.9375 ft per ft/s. Maximum height will increase by approximately $0.9375\Delta v$ feet.

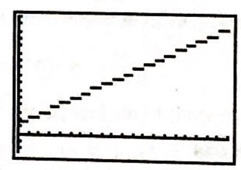
50. $4\pi\Delta v$ m^2. Area will increase by approximately $4\pi\Delta v$ m^2.

51. (a) Perhaps this is the number of bears placed in the reserve when it wa established.

(b) 200

(c) Perhaps this is the maximum number of bears that the reserve can support due to limitations of food, space, or other resources. Or, perhaps the number is capped at 200 and excess bears are moved to other locations.

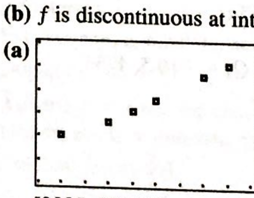
52. (a) $f(x) = \begin{cases} 3.20 - 1.35 \cdot \text{int}(-x + 1), & 0 < x \le 20 \\ 0, & x = 0 \end{cases}$



[0, 20] by [–5, 32]

(b) f is discontinuous at integer values of x: 0, 1, 2, . . . , 19

53. (a)



[2005, 2014] by [17,000, 20,000]

(b) Slope of $PQ_1 = 173$; slope of $PQ_2 = 217$; slope of $PQ_3 \approx 219.1$

(c) Answers are the same as in part (b) but with *thousand people per year* added.

(d) Answers will vary.

(e) Answers will vary.

54. $\lim_{x\to c} f(x) = 3/2$; $\lim_{x\to c} g(x) = 1/2$

55. (a) All real numbers except 3 or -3

(b) $x = -3$ and $x = 3$

(c) $y = 0$

(d) Odd, because $f(-x) = \frac{-x}{|(-x)^2 - 9|} = -\frac{x}{|x^2 - 9|} = -f(x)$ for all x in the domain.

(e) $x = -3$ and $x = 3$. Both are nonremovable.

56. **(a)** $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x^2 - a^2x) = 4 - 2a^2$.

(b) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(4 - 2x^2) = -4$

(c) For $\lim_{x\to 2} f(x)$ to exist, we must have $4 - 2a^2 = -4$, so $a = \pm 2$. If $a = \pm 2$, then $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2) = -4$, making f continuous at 2 by definition.

57. **(a)** The zeros of $f(x) = \dfrac{x^3 - 2x^2 + 1}{x^2 + 3}$ are the same as the zeros of the polynomial $x^3 - 2x^2 + 1$. By inspection, one such zero is $x = 1$. Divide $x^3 - 2x^2 + 1$ by $x - 1$ to get $x^2 - x - 1$, which has zeros $\dfrac{1 \pm \sqrt{5}}{2}$ by the quadratic formula. Thus, the zeros of f are 1, $\dfrac{1 + \sqrt{5}}{2}$, and $\dfrac{1 - \sqrt{5}}{2}$.

(b) $g(x) = x$

(c) $\lim_{x\to\infty} f(x) = +\infty$ and $\lim_{x\to\infty} \dfrac{f(x)}{g(x)} = \lim_{x\to\infty} \dfrac{x^3 - 2x^2 + 1}{x^3 + 3x} = 1$.

CHAPTER 3

Section 3.1

Quick Review 3.1

1. 4 **3.** −1 **5.** 0 **7.** $\lim_{x\to 1^+} f(x) = 0$; $\lim_{x\to 1^-} f(x) = 3$

9. No, the two one-sided limits are different.

Exercises 3.1

1. −1/4 **3.** 2 **5.** −1/4 **7.** 1/4 **9.** $f'(x) = 3$ **11.** $2x$ **13.** (b)

15. (d) **17.** **(a)** $y = 5x - 7$ **(b)** $y = -\dfrac{1}{5}x + \dfrac{17}{5}$

19. **(a)** $y = 3x - 2$ **(b)** $y = -\dfrac{1}{3}x + \dfrac{4}{3}$

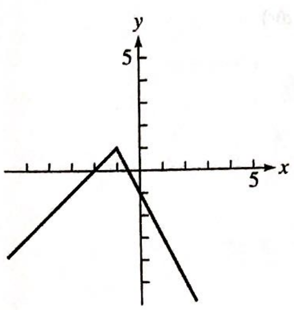
21. **(a)** Sometime around April 1. The rate then is approximately 1/6 hour per day.

(b) Yes. Jan. 1 and July 1

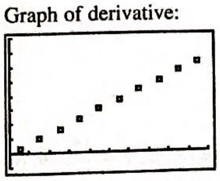
(c) Positive: Jan 1 through July 1 Negative: July 1 through Dec. 31

23. **(a)** 0, 0 **(b)** 120,000; 60,000 **(c)** 2 **25.** (iv)

27.



29. Graph of derivative:



[0,10] by [−10,80]

(a) The speed of the skier

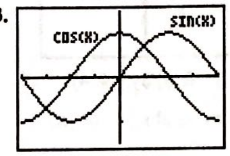
(b) Feet per second

(c) Approximately $D = 6.65t$

31. We show that the right-hand derivative at $x = 1$ does not exist:

$$\lim_{h\to 0^+} \frac{f(1 + h) - f(1)}{h} = \lim_{h\to 0^+} \frac{3(1 + h) - 2 - 2}{h} = \lim_{h\to 0^+} \frac{3h - 1}{h} = -\infty$$

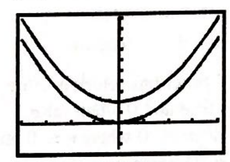
33.



$[-\pi, \pi]$ by $[-1.5, 1.5]$

Cosine could be the derivative of sine. The values of cosine are positive where sine is increasing, zero where sine has horizontal tangents, and negative where sine is decreasing.

35. Two parabolas are parallel if they have the same derivative at every value of x. This means their tangent lines are parallel at each value of x. Two such parabolas are given by $y = x^2$ and $y = x^2 + 4$. They are graphed below.



[−4, 4] by [−5, 20]

The parabolas are "everywhere equidistant," as long as the distance between them is always measured along a vertical line.

37. False. Let $f(x) = |x|$. The left-hand derivative at $x = 0$ is −1 and the right-hand derivative at $x = 0$ is 1. $f'(0)$ does not exist.

39. A **41.** C

43. **(e)** The y-intercept is $b - a$.

45. **(a)** 0.992 **(b)** 0.008

(c) If P is the answer to (b), then the probability of a shared birthday when there are four people is

$$1 - (1 - P)\frac{362}{365} \approx 0.016.$$

(d) No. Clearly, February 29th is a much less likely birth date. Furthermore, census data do not support the assumption that the other 365 birth dates are equally likely. However, this simplifying assumption may still give us some insight into this problem even if the calculated probabilities aren't completely accurate.

Section 3.2

Quick Review 3.2

1. Yes **3.** Yes **5.** No **7.** $[0, \infty)$ **9.** 3.2

Exercises 3.2

1. Left-hand derivative = 0
Right-hand derivative = 1

3. Left-hand derivative $= \dfrac{1}{2}$
Right-hand derivative = 2

5. **(a)** All points in $[-3, 2]$ **(b)** None **(c)** None

7. **(a)** All points in $[-3, 3]$ except $x = 0$ **(b)** None **(c)** $x = 0$

9. **(a)** All points in $[-1, 2]$ except $x = 0$ **(b)** $x = 0$ **(c)** None